We can often write a result as a single radical expression by finding a common denominator in the exponents.

EXAMPLE 9 Divide and, if possible, simplify: $\dfrac{\sqrt[3]{a^2b^4}}{\sqrt{ab}}$.

SOLUTION

$$\begin{aligned}
\frac{\sqrt[3]{a^2b^4}}{\sqrt{ab}} &= \frac{(a^2b^4)^{1/3}}{(ab)^{1/2}} && \text{Converting to exponential notation}\\
&= \frac{a^{2/3}b^{4/3}}{a^{1/2}b^{1/2}} && \text{Using the product and power rules}\\
&= a^{2/3-1/2}b^{4/3-1/2} && \text{Subtracting exponents}\\
&= a^{1/6}b^{5/6}\\
&= \sqrt[6]{a}\,\sqrt[6]{b^5} && \text{Converting to radical notation}\\
&= \sqrt[6]{ab^5} && \text{Using the product rule for radicals}
\end{aligned}$$

Try Exercise 93.

7.5 Exercise Set

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Concept Reinforcement *For each of Exercises 1–6, fill in the blanks by selecting from the following words (which may be used more than once):*

radicand(s), indices, conjugate(s), base(s), denominator(s), numerator(s).

1. To add radical expressions, the __radicands__ and the __indices__ must be the same.
2. To multiply radical expressions, the __indices__ must be the same.
3. To find a product by adding exponents, the __bases__ must be the same.
4. To add rational expressions, the __denominators__ must be the same.
5. To rationalize the __numerator__ of $\dfrac{\sqrt{c}-\sqrt{a}}{5}$, we multiply by a form of 1, using the __conjugate__ of $\sqrt{c}-\sqrt{a}$, or $\sqrt{c}+\sqrt{a}$, to write 1.
6. To find a quotient by subtracting exponents, the __bases__ must be the same.

Add or subtract. Simplify by combining like radical terms, if possible. Assume that all variables and radicands represent positive real numbers.

7. $2\sqrt{5}+7\sqrt{5}$ $9\sqrt{5}$
8. $4\sqrt{7}+2\sqrt{7}$ $6\sqrt{7}$
9. $7\sqrt[3]{4}-5\sqrt[3]{4}$ $2\sqrt[3]{4}$
10. $14\sqrt[5]{2}-6\sqrt[5]{2}$ $8\sqrt[5]{2}$
11. $\sqrt[3]{y}+9\sqrt[3]{y}$ $10\sqrt[3]{y}$
12. $9\sqrt[4]{t}-\sqrt[4]{t}$ $8\sqrt[4]{t}$
13. $8\sqrt{2}-\sqrt{2}+5\sqrt{2}$ $12\sqrt{2}$
14. $\sqrt{6}-8\sqrt{6}+2\sqrt{6}$ $-5\sqrt{6}$
15. $9\sqrt[3]{7}-\sqrt{3}+4\sqrt[3]{7}+2\sqrt{3}$ $13\sqrt[3]{7}+\sqrt{3}$
16. $5\sqrt{7}-8\sqrt[4]{11}+\sqrt{7}+9\sqrt[4]{11}$ $6\sqrt{7}+\sqrt[4]{11}$
17. $4\sqrt{27}-3\sqrt{3}$ $9\sqrt{3}$
18. $9\sqrt{50}-4\sqrt{2}$ $41\sqrt{2}$
19. $3\sqrt{45}-8\sqrt{20}$ $-7\sqrt{5}$
20. $5\sqrt{12}+16\sqrt{27}$ $58\sqrt{3}$

21. $3\sqrt[3]{16} + \sqrt[3]{54}$ $9\sqrt[3]{2}$
22. $\sqrt[3]{27} - 5\sqrt[3]{8}$ -7
23. $\sqrt{a} + 3\sqrt{16a^3}$ $(1 + 12a)\sqrt{a}$
24. $2\sqrt{9x^3} - \sqrt{x}$ $(6x - 1)\sqrt{x}$
25. $\sqrt[3]{6x^4} - \sqrt[3]{48x}$ $(x - 2)\sqrt[3]{6x}$
26. $\sqrt[3]{54x} - \sqrt[3]{2x^4}$ $(3 - x)\sqrt[3]{2x}$
27. $\sqrt{4a - 4} + \sqrt{a - 1}$ $3\sqrt{a - 1}$
28. $\sqrt{9y + 27} + \sqrt{y + 3}$ $4\sqrt{y + 3}$
29. $\sqrt{x^3 - x^2} + \sqrt{9x - 9}$ $(x + 3)\sqrt{x - 1}$
30. $\sqrt{4x - 4} - \sqrt{x^3 - x^2}$ $(2 - x)\sqrt{x - 1}$

Multiply. Assume that all variables represent nonnegative real numbers.

31. $\sqrt{3}(4 + \sqrt{3})$ $4\sqrt{3} + 3$
32. $\sqrt{7}(3 - \sqrt{7})$ $3\sqrt{7} - 7$
33. $3\sqrt{5}(\sqrt{5} - \sqrt{2})$ ⊡
34. $4\sqrt{2}(\sqrt{3} - \sqrt{5})$ ⊡
35. $\sqrt{2}(3\sqrt{10} - 2\sqrt{2})$ ⊡
36. $\sqrt{3}(2\sqrt{5} - 3\sqrt{4})$ ⊡
37. $\sqrt[3]{3}(\sqrt[3]{9} - 4\sqrt[3]{21})$ ⊡
38. $\sqrt[3]{2}(\sqrt[3]{4} - 2\sqrt[3]{32})$ ⊡
39. $\sqrt[3]{a}(\sqrt[3]{a^2} + \sqrt[3]{24a^2})$ $a + 2a\sqrt[3]{3}$
40. $\sqrt[3]{x}(\sqrt[3]{3x^2} - \sqrt[3]{81x^2})$ $-2x\sqrt[3]{3}$
41. $(2 + \sqrt{6})(5 - \sqrt{6})$ $4 + 3\sqrt{6}$
42. $(4 - \sqrt{5})(2 + \sqrt{5})$ $3 + 2\sqrt{5}$
43. $(\sqrt{2} + \sqrt{7})(\sqrt{3} - \sqrt{7})$ $\sqrt{6} - \sqrt{14} + \sqrt{21} - 7$
44. $(\sqrt{7} - \sqrt{2})(\sqrt{5} + \sqrt{2})$ $\sqrt{35} + \sqrt{14} - \sqrt{10} - 2$
45. $(3 - \sqrt{5})(3 + \sqrt{5})$ 4
46. $(2 + \sqrt{11})(2 - \sqrt{11})$ -7
47. $(\sqrt{6} + \sqrt{8})(\sqrt{6} - \sqrt{8})$ -2
48. $(\sqrt{12} - \sqrt{7})(\sqrt{12} + \sqrt{7})$ 5
49. $(3\sqrt{7} + 2\sqrt{5})(2\sqrt{7} - 4\sqrt{5})$ $2 - 8\sqrt{35}$
50. $(4\sqrt{5} - 3\sqrt{2})(2\sqrt{5} + 4\sqrt{2})$ $16 + 10\sqrt{10}$
51. $(2 + \sqrt{3})^2$ $7 + 4\sqrt{3}$
52. $(3 + \sqrt{7})^2$ $16 + 6\sqrt{7}$
53. $(\sqrt{3} - \sqrt{2})^2$ $5 - 2\sqrt{6}$
54. $(\sqrt{5} - \sqrt{3})^2$ $8 - 2\sqrt{15}$
55. $(\sqrt{2t} + \sqrt{5})^2$ $2t + 5 + 2\sqrt{10t}$
56. $(\sqrt{3x} - \sqrt{2})^2$ $3x + 2 - 2\sqrt{6x}$
57. $(3 - \sqrt{x + 5})^2$ $14 + x - 6\sqrt{x + 5}$
58. $(4 + \sqrt{x - 3})^2$ $13 + x + 8\sqrt{x - 3}$
59. $(2\sqrt[4]{7} - \sqrt[4]{6})(3\sqrt[4]{9} + 2\sqrt[4]{5})$ ⊡
60. $(4\sqrt[3]{3} + \sqrt[3]{10})(2\sqrt[3]{7} + 5\sqrt[3]{6})$ ⊡

Rationalize each denominator.

61. $\dfrac{6}{3 - \sqrt{2}}$ $\dfrac{18 + 6\sqrt{2}}{7}$
62. $\dfrac{3}{4 - \sqrt{7}}$ $\dfrac{4 + \sqrt{7}}{3}$
63. $\dfrac{2 + \sqrt{5}}{6 + \sqrt{3}}$ ⊡
64. $\dfrac{1 + \sqrt{2}}{3 + \sqrt{5}}$ ⊡
65. $\dfrac{\sqrt{a}}{\sqrt{a} + \sqrt{b}}$ $\dfrac{a - \sqrt{ab}}{a - b}$
66. $\dfrac{\sqrt{z}}{\sqrt{x} - \sqrt{z}}$ $\dfrac{z + \sqrt{xz}}{x - z}$

Aha! 67. $\dfrac{\sqrt{7} - \sqrt{3}}{\sqrt{3} - \sqrt{7}}$ -1

68. $\dfrac{\sqrt{7} + \sqrt{5}}{\sqrt{5} + \sqrt{2}}$ ⊡
69. $\dfrac{3\sqrt{2} - \sqrt{7}}{4\sqrt{2} + 2\sqrt{5}}$ ⊡
70. $\dfrac{5\sqrt{3} - \sqrt{11}}{2\sqrt{3} - 5\sqrt{2}}$ ⊡

Rationalize each numerator. If possible, simplify your result.

71. $\dfrac{\sqrt{5} + 1}{4}$ $\dfrac{1}{\sqrt{5} - 1}$
72. $\dfrac{\sqrt{3} + 1}{4}$ $\dfrac{1}{2\sqrt{3} - 2}$
73. $\dfrac{\sqrt{6} - 2}{\sqrt{3} + 7}$ ⊡
74. $\dfrac{\sqrt{10} + 4}{\sqrt{2} - 3}$ ⊡
75. $\dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$ $\dfrac{x - y}{x + 2\sqrt{xy} + y}$
76. $\dfrac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}$ ⊡
77. $\dfrac{\sqrt{a + h} - \sqrt{a}}{h}$ ⊡
78. $\dfrac{\sqrt{x - h} - \sqrt{x}}{h}$ ⊡

Perform the indicated operation and simplify. Assume that all variables represent positive real numbers.

79. $\sqrt[3]{a}\,\sqrt[6]{a}$ $\sqrt{a}$
80. $\sqrt[10]{a}\,\sqrt[5]{a^2}$ $\sqrt{a}$
81. $\sqrt[5]{b^2}\,\sqrt{b^3}$ $b\sqrt[10]{b^9}$
82. $\sqrt[4]{a^3}\,\sqrt[3]{a^2}$ $a\sqrt[12]{a^5}$
83. $\sqrt{xy^3}\,\sqrt[3]{x^2y}$ $xy\sqrt[6]{xy^5}$
84. $\sqrt[5]{a^3b}\,\sqrt{ab}$ $a\sqrt[10]{ab^7}$
85. $\sqrt[4]{9ab^3}\,\sqrt{3a^4b}$ $3a^2b\sqrt[4]{ab}$
86. $\sqrt{2x^3y^3}\,\sqrt[3]{4xy^2}$ ⊡
87. $\sqrt{a^4b^3c^4}\,\sqrt[3]{ab^2c}$ ⊡
88. $\sqrt[3]{xy^2z}\,\sqrt{x^3yz^2}$ ⊡
89. $\dfrac{\sqrt[3]{a^2}}{\sqrt[4]{a}}$ $\sqrt[12]{a^5}$
90. $\dfrac{\sqrt[3]{x^2}}{\sqrt[5]{x}}$ $\sqrt[15]{x^7}$
91. $\dfrac{\sqrt[4]{x^2y^3}}{\sqrt[3]{xy}}$ $\sqrt[12]{x^2y^5}$
92. $\dfrac{\sqrt[5]{a^4b}}{\sqrt[3]{ab}}$ $\sqrt[15]{\dfrac{a^7}{b^2}}$
93. $\dfrac{\sqrt{ab^3}}{\sqrt[5]{a^2b^3}}$ $\sqrt[10]{ab^9}$
94. $\dfrac{\sqrt[5]{x^3y^4}}{\sqrt{xy}}$ $\sqrt[10]{xy^3}$
95. $\dfrac{\sqrt{(7 - y)^3}}{\sqrt[3]{(7 - y)^2}}$ $\sqrt[6]{(7 - y)^5}$
96. $\dfrac{\sqrt[5]{(y - 9)^3}}{\sqrt{y - 9}}$ $\sqrt[10]{y - 9}$

⊡ Answers to Exercises 33–38, 59, 60, 63, 64, 68–70, 73, 74, 76–78, and 86–88 are on p. IA-16.

97. $\dfrac{\sqrt[4]{(5+3x)^3}}{\sqrt[3]{(5+3x)^2}}$ $\sqrt[12]{5+3x}$

98. $\dfrac{\sqrt[3]{(2x+1)^2}}{\sqrt[5]{(2x+1)^2}}$ $\sqrt[15]{(2x+1)^4}$

99. $\sqrt[3]{x^2y}(\sqrt{xy}-\sqrt[5]{xy^3})$ $x\sqrt[6]{xy^5}-\sqrt[15]{x^{13}y^{14}}$

100. $\sqrt[4]{a^2b}(\sqrt[3]{a^2b}-\sqrt[5]{a^2b^2})$ $a\sqrt[12]{a^2b^7}-\sqrt[20]{a^{18}b^{13}}$

101. $(m+\sqrt[3]{n^2})(2m+\sqrt[4]{n})$ ⊡

102. $(r-\sqrt[4]{s^3})(3r-\sqrt[5]{s})$ ⊡

In Exercises 103–106, $f(x)$ and $g(x)$ are as given. Find $(f \cdot g)(x)$. Assume that all variables represent nonnegative real numbers.

103. $f(x)=\sqrt[4]{x},\ g(x)=2\sqrt{x}-\sqrt[3]{x^2}$ $2\sqrt[4]{x^3}-\sqrt[12]{x^{11}}$

104. $f(x)=\sqrt[5]{x}+5\sqrt{x},\ g(x)=\sqrt[3]{x^2}$ $\sqrt[15]{x^{13}}+5x\sqrt[6]{x}$

105. $f(x)=x+\sqrt{7},\ g(x)=x-\sqrt{7}$ x^2-7

106. $f(x)=x-\sqrt{2},\ g(x)=x+\sqrt{6}$ $x^2+x\sqrt{6}-x\sqrt{2}-2\sqrt{3}$

Let $f(x)=x^2$. Find each of the following.

107. $f(5+\sqrt{2})$ $27+10\sqrt{2}$

108. $f(7+\sqrt{3})$ $52+14\sqrt{3}$

109. $f(\sqrt{3}-\sqrt{5})$ $8-2\sqrt{15}$

110. $f(\sqrt{6}-\sqrt{3})$ $9-6\sqrt{2}$

111. In what way(s) is combining like radical terms similar to combining like terms that are monomials?

TW 112. Why do we need to know how to multiply radical expressions before learning how to add them?

SKILL REVIEW

To prepare for Section 7.6, review solving equations (Sections 1.6, 5.4, 5.5, and 6.4).

Solve.

113. $3x-1=125$ [1.6] 42

114. $x+5-2x=3x+6-x$ [1.6] $-\frac{1}{3}$

115. $x^2+2x+1=22-2x$ [5.4] $-7, 3$

116. $9x^2-6x+1=7+5x-x^2$ [5.5] $-\frac{2}{5}, \frac{3}{2}$

117. $\dfrac{1}{x}+\dfrac{1}{2}=\dfrac{1}{6}$ [6.4] -3

118. $\dfrac{x}{x-4}+\dfrac{2}{x+4}=\dfrac{x-2}{x^2-16}$ [6.4] $-6, 1$

SYNTHESIS

TW 119. Ramon *incorrectly* writes

$$\sqrt[5]{x^2}\cdot\sqrt{x^3}=x^{2/5}\cdot x^{3/2}=\sqrt[5]{x^3}.$$

What mistake do you suspect he is making?

TW 120. After examining the expression $\sqrt[4]{25xy^3}\sqrt{5x^4y}$, Marika (correctly) concludes that both x and y are nonnegative. Explain how she could reach this conclusion.

Find a simplified form for $f(x)$. Assume $x \geq 0$.

121. $f(x)=\sqrt{x^3-x^2}+\sqrt{9x^3-9x^2}-\sqrt{4x^3-4x^2}$ ⊡

122. $f(x)=\sqrt{20x^2+4x^3}-3x\sqrt{45+9x}+\sqrt{5x^2+x^3}$ ⊡

123. $f(x)=\sqrt[4]{x^5-x^4}+3\sqrt[4]{x^9-x^8}$ ⊡

124. $f(x)=\sqrt[4]{16x^4+16x^5}-2\sqrt[4]{x^8+x^9}$ ⊡

Simplify.

125. $7x\sqrt{(x+y)^3}-5xy\sqrt{x+y}-2y\sqrt{(x+y)^3}$ ⊡

126. $\sqrt{27a^5(b+1)}\sqrt[3]{81a(b+1)^4}$ ⊡

127. $\sqrt{8x(y+z)^5}\sqrt[3]{4x^2(y+z)^2}$ ⊡

128. $\frac{1}{2}\sqrt{36a^5bc^4}-\frac{1}{2}\sqrt[3]{64a^4bc^6}+\frac{1}{6}\sqrt{144a^3bc^6}$ ⊡

129. $\dfrac{\dfrac{1}{\sqrt{w}}-\sqrt{w}}{\dfrac{\sqrt{w}+1}{\sqrt{w}}}$ $1-\sqrt{w}$

130. $\dfrac{1}{4+\sqrt{3}}+\dfrac{1}{\sqrt{3}}+\dfrac{1}{\sqrt{3}-4}$ $\dfrac{7\sqrt{3}}{39}$

Express each of the following as the product of two radical expressions.

131. $x-5$ $(\sqrt{x}+\sqrt{5})(\sqrt{x}-\sqrt{5})$

132. $y-7$ $(\sqrt{y}+\sqrt{7})(\sqrt{y}-\sqrt{7})$

133. $x-a$ $(\sqrt{x}+\sqrt{a})(\sqrt{x}-\sqrt{a})$

Multiply.

134. $\sqrt{9+3\sqrt{5}}\sqrt{9-3\sqrt{5}}$ 6

135. $(\sqrt{x+2}-\sqrt{x-2})^2$ $2x-2\sqrt{x^2-4}$

Try Exercise Answers: Section 7.5

7. $9\sqrt{5}$ 17. $9\sqrt{3}$ 33. $15-3\sqrt{10}$ 61. $\dfrac{18+6\sqrt{2}}{7}$ 71. $\dfrac{1}{\sqrt{5}-1}$ 79. $\sqrt{a}$ 89. $\sqrt[12]{a^5}$ 93. $\sqrt[10]{ab^9}$ 103. $2\sqrt[4]{x^3}-\sqrt[12]{x^{11}}$

⊡ Answers to Exercises 101, 102, and 121–128 are on p. IA-16.